Enrollment No. ______________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous), Rajkot
Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION NOVEMBER - 2017

M.Sc. Mathematics

16PMTCC13 - NUMBER THEORY-I

Duration of Exam – 3 hrs ***Semester – III*** ***Max. Marks – 70***

***Part A** (5X2=10 marks)*

Answer **ALL** questions

1. Is ‘0’ even or odd or neither? Justify your answer.
2. Define multiplicative function.
3. State well ordering principle.
4. State Fermat’s Theorem.
5. Is 6 and 15 relatively prime? Justify your answer.

***Part B** (5X5= 25 marks)*

Answer **ALL** questions

6a. If $g=(a,b)$ then prove that there exists integers x_0 and y_0 such that $g=ax_0+by_0$.

OR

6b. If a,b,c are integers with $a\neq 0$, $b\neq 0$, $c\neq 0$ then prove that $(a,b,c)=((a,b),c)=(a,(b,c))=((a,c),b)$.

7a. State and prove Euclidean Algorithm theorem.

OR

7b. If $(a,m)=1$ and $(b,m)=1$ then prove that $(ab,m)=1$.

8a. Let x and y be real numbers. Then prove that

(i) $[x+m]=[x]+m$ if m is an integer.

(ii) $[x]+[y]\leq[x+y]\leq[x]+[y]+1$.

OR

8b. If $n = p_1^{e_1} p_2^{e_2} \ldots p_r^{e_r}$ then prove that ² $(n)=(e_1+1)(e_2+1)\ldots(e_r+1)$.

9a. State and prove Euler’s Theorem.

OR

9b. Let $l=[a,b]$ and m is a common positive multiple of a and b then prove that l/m.

10a. If m/ab then prove that $\frac{m}{(a,m)}/b$.

OR

10b. State and prove Chinese Remainder Theorem.

Part C ***(5X7= 35 marks)***

Answer **ALL** questions

11a. State and prove Division Algorithm Theorem.

OR

11b. Let a and b are integers with $a \neq 0$ and $m \neq 0$. Then the congruence equation $f(x) = ax - b \equiv 0 \pmod{m}$ has a solution if and only if $(a,m)/b$.

12a. State and prove Hansel’s Lemma.

OR

12b. Let p be a prime number, $d \geq 1$. If $d/p-1$ then $x^d - 1 \equiv 0 \pmod{p}$ have exactly d solutions in any complete residue system (mod p).

13a. Let $m \geq 1$, $(a,m) = 1$ and order of $a (\bmod m) = h$. If k is a positive integer such that $a^k \equiv 1 \pmod{m}$ then prove that h divides k.

OR

13b. State and prove Wilsons Theorem.

14a. Let $m \geq 1$ and $m = m_1 m_2$ with $m_1 m_2 \geq 1$, $(m_1, m_2) = 1$. If $(¤(m_1), ¨(m_2)) \geq 2$ then m does not have a primitive root.

OR

14b. State and prove Fundamental Theorem of Arithmetic.

15a. Let $m \geq 1$ and order of $a(\bmod m) = h$ and $k \geq 1$ then prove that the order of $a^k (\bmod m) = \frac{h}{(h,k)}$.

OR

15b. Let p denote a prime. Then the largest exponent e such that $p^e / n!$ is $e = \sum_{i=1}^{\infty} \left[\frac{n}{p^i} \right]$.
